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DEPARTMENT OF ELECTRICAL ENGINEERING

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OCEAN-EARTH ACOUSTIC COUPLING

D. K. Reynolds
G. L. Mitchell
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30 November 1976

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Prepared for

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Experimental measurements to investigate the coupling of guided acoustic waves from a liquid layer of tapered depth into a solid substrate were undertaken. Results indicate that efficient coupling from water into a solid material modeling hard rock occurs principally at depths of less than one wavelength. Higher order locally normal modes were found to be converted to lower order modes without significant substrate radiation. The lowest order mode radiated into the solid as a well defined beam.

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ABSTRACT

Experimental measurements to investigate the coupling of guided acoustic waves from a liquid layer of tapered depth into a solid substrate were undertaken. Results indicate that efficient coupling from water to a solid material modeling hard rock occurs principally at depths of less than one wavelength. Higher order locally normal modes were found to be converted to lower order modes without significant substrate radiation. Radiation in the solid was a well defined shear wave beam.

INTRODUCTION

The objective of this research is the study of the coupling of low frequency acoustic waves from the ocean into the underlying earth, as the acoustic waves approach a shoreline. While acoustic wave propagation has been extensively studied in uniform layered media, including the case of a layer of water over a solid elastic half-space, no studies appear to have been made for the case of a layer of water of tapered depth, approximating shoreline geometry.

Acoustic waves in a layer of water of uniform thickness above an elastic solid can be analyzed in terms of a superposition of normal modes. In a very gradually tapered layer, whose slope is less than 1 part of 100, one may expect that at any location the modal structure will conform closely to that of a uniform layer of corresponding depth. Since the acoustic wave modes exhibit cut off phenomena, of particular interest are the mode conversions as successive modes reach cut off depth. As a given mode is cut off, the question arises as to how much energy in that mode is converted into lower order propagating modes, how much is reflected, and how much is radiated into the underlying solid. The principal objective of this project has been to investigate these questions through a largely experimental approach.

In choosing an appropriate model for laboratory experiments, initial thought was given to the use of electromagnetic waves at both microwave and optical frequencies. This approach was abandoned in favor of acoustic modeling, principally because the electromagnetic model can not properly represent the superposition of transverse and longitudinal waves, both of which are present in the acoustic case. Furthermore, through the use of ultrasonic waves, a compact, inexpensive laboratory setup was possible. Using ultrasonic waves in the 1 - 2.5 MHz range, schlieren techniques were

initially used to obtain qualitative results on energy coupling into the solid. Encouraging results from the schlieren measurements led to the construction of a much larger tank which allowed the shoreline geometry to be more accurately modeled, and quantitative data to be taken using probes and pulse techniques.

As well as explained in this report, the technique used to measure the location, magnitude, and direction of energy coupled from the tapered layer of water into the solid was to use a slab of solid material immersed in water, and detect the energy coupled through the solid into the water below. From the angle of the emerging beam and its location, the region of energy coupling in the water above the slab could be inferred. The measured magnitude of the emerging beam was used to estimate the efficiency of the coupling mechanism.

Results of the experiments, described in detail below, may be summarized as follows:

1. Energy coupling for the case of water over an aluminum slab (modeling hard rock) appears to be localized in the region where the lowest order mode has reached cut off (water depth less than one wavelength).
2. The energy coupled out from the solid into the water below, and hence the energy within the solid, is in the form of a well defined beam.
3. The efficiency of coupling is high. The intensity of the acoustic field in the solid in the region of maximum coupling has been calculated from experimental data to be only 7 dB below the level measured in free water with the same probe to source separation.

An important contribution of this program has been the development of relatively inexpensive modeling techniques and experimental methods to study a broad class of underwater acoustic problems.

Recommendations for future work, suggested by the results of this investigation include:

1. Theoretical studies of mode coupling in the tapered region. This will be essential for the interpretation of future experimental work.
2. More precise experimental work. The acoustic tank constructed for this project is entirely adequate for continued work, when upgraded with precision probe carriages and automated data logging capabilities.
3. Guidance to future full scale experiments. Results of theoretical studies and model measurements will lead to the specification of the most appropriate full scale measurements.

ANALYSIS

Planar Acoustic Waveguides

The simplest application of normal mode theory to shallow water acoustics is in the case of a planar waveguide as shown in Figure 1. Analysis of this structure must take acount of the characteristics of the various layers to determine reflection coefficients at the two interface planes. For guided modes on a lossless structure the reflection coefficient at each interface is purely imaginary. The resulting transcendental equation for the waveguide eigenvalue is relatively easy to solve. The characteristic equation for this waveguide, Equation 1, has been derived by Pekeris and discussed by others.[1,2]

$$\gamma_1 h + \tan^{-1}\left\{\frac{\rho_2\,\gamma_1}{\rho_1 i \gamma_2 k_{S2}} \quad 4\,\gamma_2\delta_2\alpha^2 + (\alpha^2 - \delta^2)^2\right\} = m\pi \qquad (1)$$

where

γ_j = transverse propagation constant, compression wave

δ_j = transverse propagation constant, shear wave

ρ_j = density

$i = \sqrt{-1}$

$k = 2\pi/\lambda$

λ = acoustic wavelength

Equation 1 can be modified to acount for factors such as varying acoustic velocity as a function of water depth or losses on the water or in the earth.

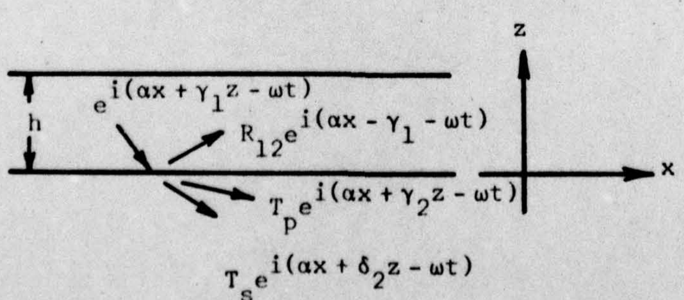

FIGURE 1. Model used for acoustic waveguide calculations.

Guided modes supported by the structure shown in Figure 1 are normal; that is, unless the material properties of the waveguide vary as a function of x guided modes will not couple from one to another. They will also not couple to the continuous spectrum of radiation modes which are associated with the waveguides. The analysis associated with Figure 1 can be used to define locally normal modes which are supported by a structure which varies as a function of x. This method of analysis presumes that the taper of the waveguide shown in Figure 2 is approximated by regions of constant depth, each represented by normal modes with eigenvalues determined by Equation 1. Since the tapered case requires that longitudinal components of the wave equation be considered, the locally normal modes do couple to one another and also the radiation spectrum.

Figure 3 from Ewing <u>et al.</u>[1] shows the cut-off of the lowest order guided mode as the normalized thickness of the guide goes to zero. In this case, the guided wave phase velocity at zero thickness corresponds to that of a Rayleigh wave. Material constants used for the derivation of Figure 1 data are approximately those of the aluminum/water guide. The same plot for a liquid-liquid waveguide, Figure 4 indicates cut-off to the radiation spectrum as thickness goes to zero. A problem which must be solved in the ocean acoustics case is the relative distribution of energy to the boundary disturbances and radiation when the guide is tapered.

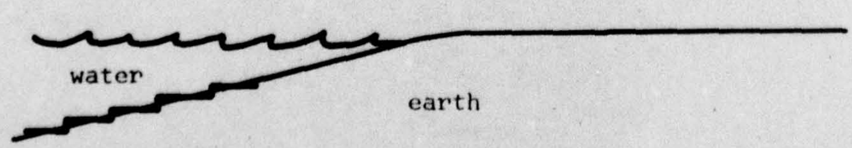

FIGURE 2. Acoustic waveguide with varying depth.

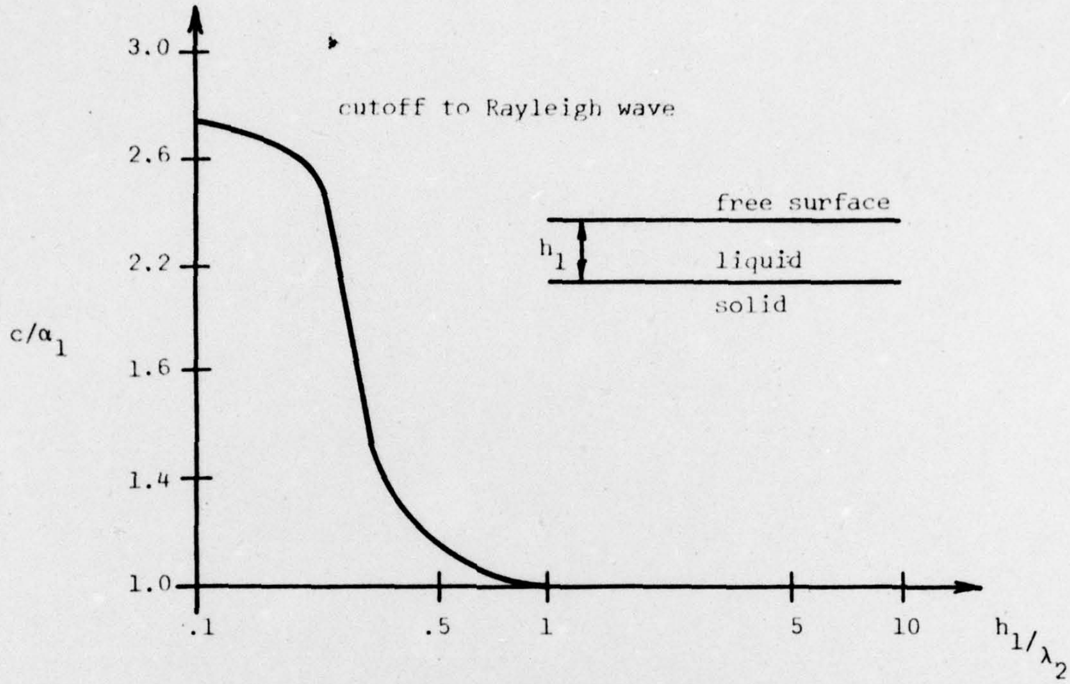

FIGURE 3. Phase velocity c in terms of substrate velocity α_1 for a liquid over solid.[1] $\rho_2/\rho_1 = 3$, $\alpha_2/\beta_2 = \sqrt{3}$, $\beta_2/\alpha_1 = 3$ where α and β are compressional and shear wave velocities.

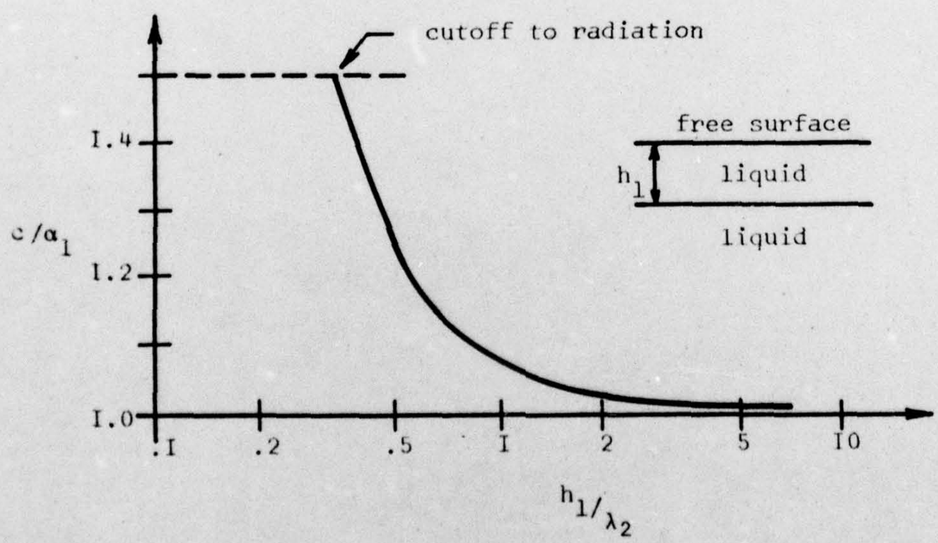

FIGURE 4. Phase velocity for a liquid over liquid waveguide.[1] $\rho_2/\rho_1 = 2$, $\alpha_2/\alpha_1 = 1.5$.

Tapered Acoustic Waveguides

Tien and Martin[3] have demonstrate with the structure shown in Figure 5 that guided electromagnetic waves in a tapered guide will be radiated in a well defined beam which propagates into the substrate of the waveguide. In this case refractive indices are shown for each layer and typical energy distributions are plotted for the waveguide and the radiated beam. The tapered film edge has a slope of 0.002. This optical example is very similar to the ocean acoustics situation except that all layers are perfectly homogeneous with no velocity variation as a function of z and nothing analogous to conversion between shear and compressional waves exists in the optical case. Figure 6 illustrates a possible ocean acoustics analog of Figure 5. In this case energy distributions both in the waveguide and in the radiated beam are more complex. Inhomogeneities in the earth will cause some scattering of the radiated beam.

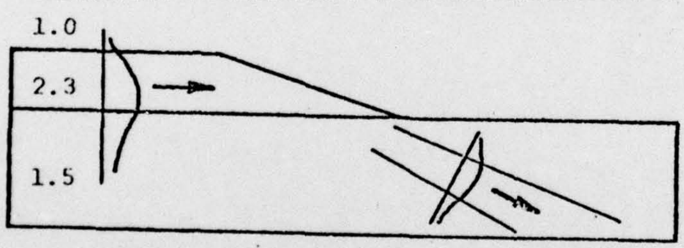

FIGURE 5. Optical tapered coupler.

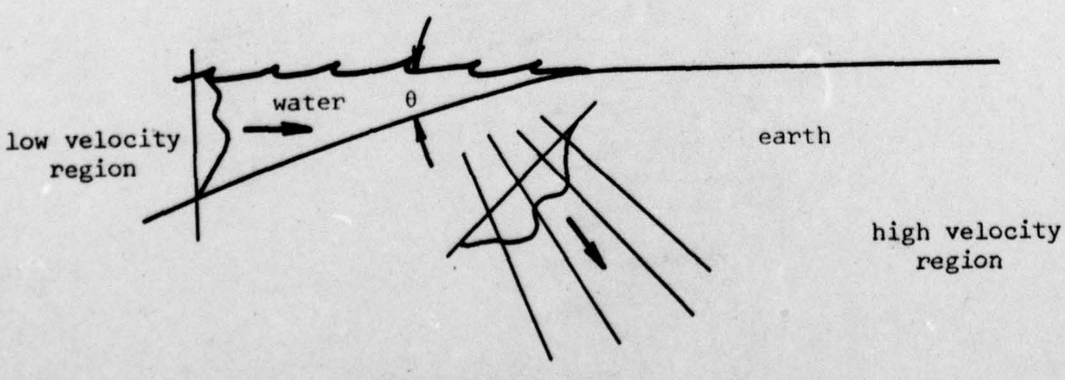

FIGURE 6. Ocean/earth tapered coupler.

Acoustic waves traveling within a water-earth waveguide can be expected to couple to lower order modes as they approach cut-off.[4] If the slope is sufficiently small negligible radiation at cut-off will occur, except for the lowest order mode (which in a symmetric waveguide cuts off at zero thickness). Before the zeroth order mode cuts off coupling to the radiation spectrum and boundary disturbances will occur. An analysis of the power loss in the waveguide by Marcuse[5] shows that power in a tapered dielectric electromagnetic guide decreases sharply a few wavelengths from the cut-off thickness and at the cut-off point virtually all of the energy in the guide has radiated. Exact definitions of the radiated field, even for the homogeneous case of Figure 5 is difficult, and could probably best be done by Green's function techniques used by Ewing[1] or the stepped planar waveguide solution.

Grating Coupler Analysis

In an attempt to increase coupling from discrete waveguide modes to the radiation spectrum some aluminum models had periodic ridges milled into the top surface. Figure 7 shows this grating matching in detail. The grating period was designed to couple between all guided modes to aluminum shear and in some cases, compressional waves; no coupling between guided modes was expected since the grating vector magnitude $|K| = 2\pi/\lambda_{grating}$ was greater than the difference between propagation vectors for a water compressional wave. This analysis follows the method used for integrated optics.

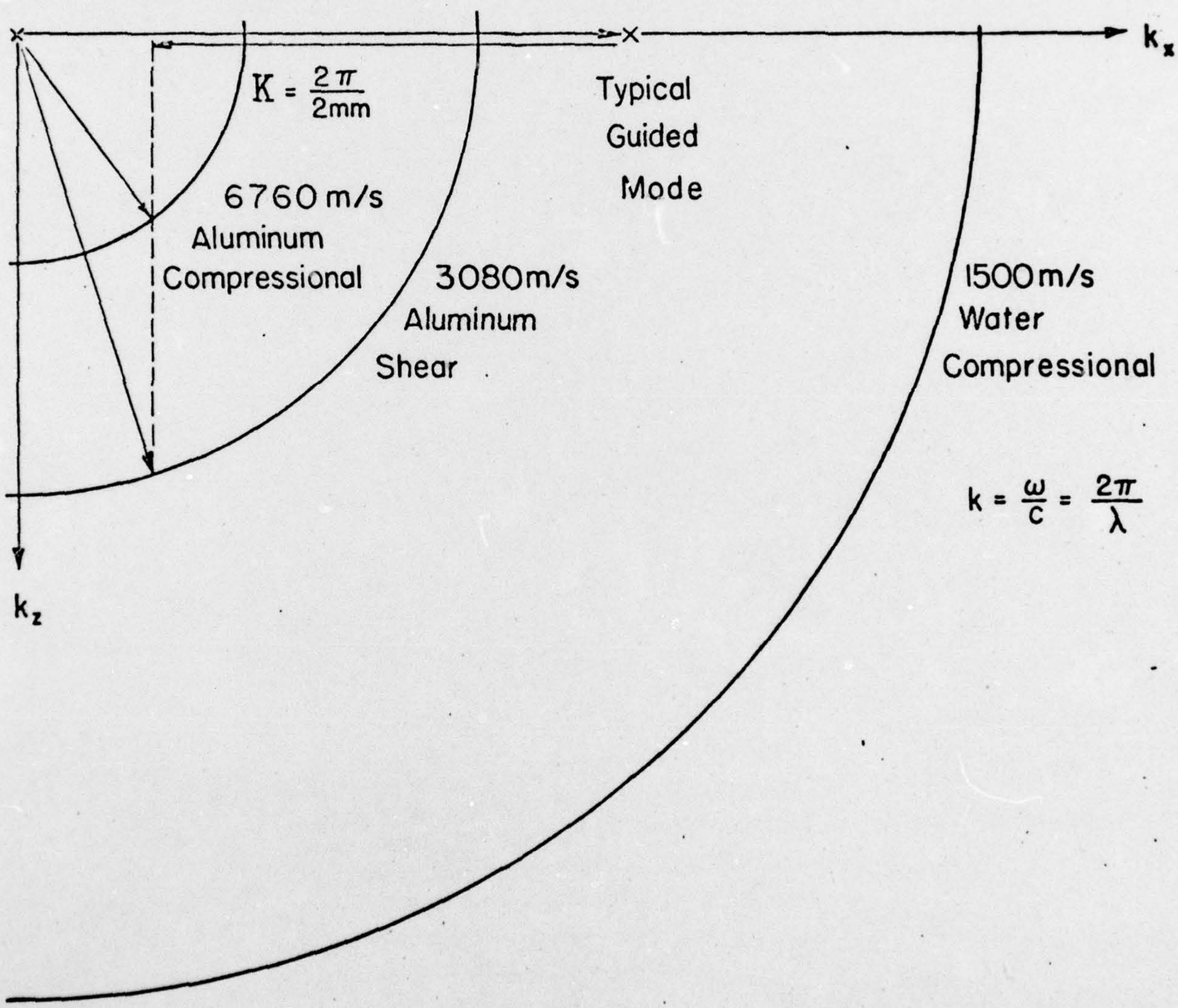

FIGURE 7. Propagation vector (phase match) plots for grating coupler design. K is the grating vector for coupling to the radiation spectrum with minimum coupling between guided modes.

DESIGN OF THE EXPERIMENT

Models and Materials

Manufacturing simplicity and material availability prompted the choice of plastic (polymethylmethacrylate or PMMA) for Schlieren models. This material has a velocity of propagation for the longitudinal (compression) wave of 2670 m/s and 1120 m/s for the transverse (shear) wave. For a water - PMMA waveguide shear waves will not be totally reflected since for any angle of incidence, a water compressional wave will create a shear wave within the PMMA. Incident waves more than 34^{o} from the normal do not couple to compressional waves in the solid, however. This means that such a waveguide loses acoustic energy continuously along its length to shear waves in the solid and, where the lowest order cut-off to compressional waves.

Models for quantitative field measurements (and some of the later Schlieren tests) were constructed from aluminum. Shear and compressional wave velocities for aluminum are respectively 3080 m/s and 6260 m/s. These values are close enough to corresponding velocities for rock to make the choice of aluminum as a material for models reasonable.[6] The water-aluminum interface is totally reflecting except when propagating modes cut off. Because the shear wave velocity is closer to water propagation velocity, most energy radiated into the aluminum is in the form of shear waves.

A 1 MHz CW source was chosen for the Schlieren tests because it was capable of producing a 25 watt acoustic output. The quantitative experiments used 2.5 MHz transducers to take advantage of an existing focused transducer as a probe.

EXPERIMENTAL RESULTS

Schlieren Photography

As a first step in analysis of the coupling problem, a plastic (PMMA) model was used as an acoustic model Figures 8 and 9 show the Schlieren setup used and Figure 10 the model with output beams indicated. In the plastic model, water guided acoustic modes have a phase velocity of about 1500 m/s; the shear and compressional wave velocities for PMMA are 1120 m/s and 2760 m/s. This allows guiding of a wave in the water with constant leakage to the plastic shear wave, with cut-off of all guided energy to the compressional wave as the waveguide thickness approaches zero. Output beams, when traced back through the water-plastic interface indicate that coupling is primarily from the shallow portion of the waveguide.

Aluminum models were constructed to provide more representative velocities for aluminum v_s = 3080 m/s and v_c = 6260 m/s, a much better match to the propagation conditions found in the earth. Figures 11 and 12 show the aluminum models which were evaluated. Reasonably efficient coupling from guided modes to the (aluminum) substrate was observed. Details of the setup were shown in Figure 9. In addition to the smooth models, one was constructed with a grating coupling structure on the top surface. This grating was designed to optimize coupling between bound modes in the guide and the radiation spectrum. For the materials and acoustic wavelength used a grating period of 2 mm was optimum. (See Figure 7.) No grating effects were observed, however.

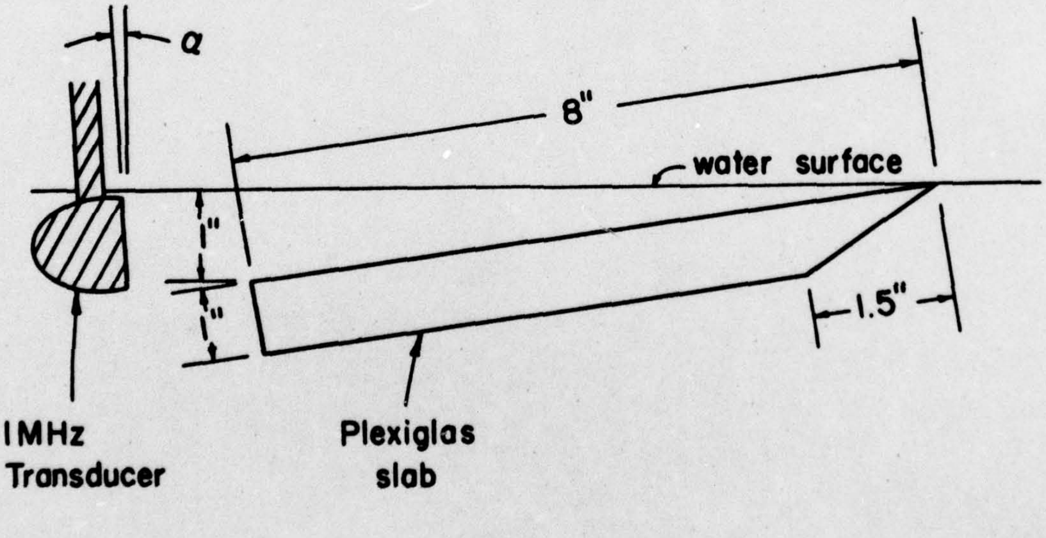

FIGURE 8. Geometry of the schlieren experimental model.

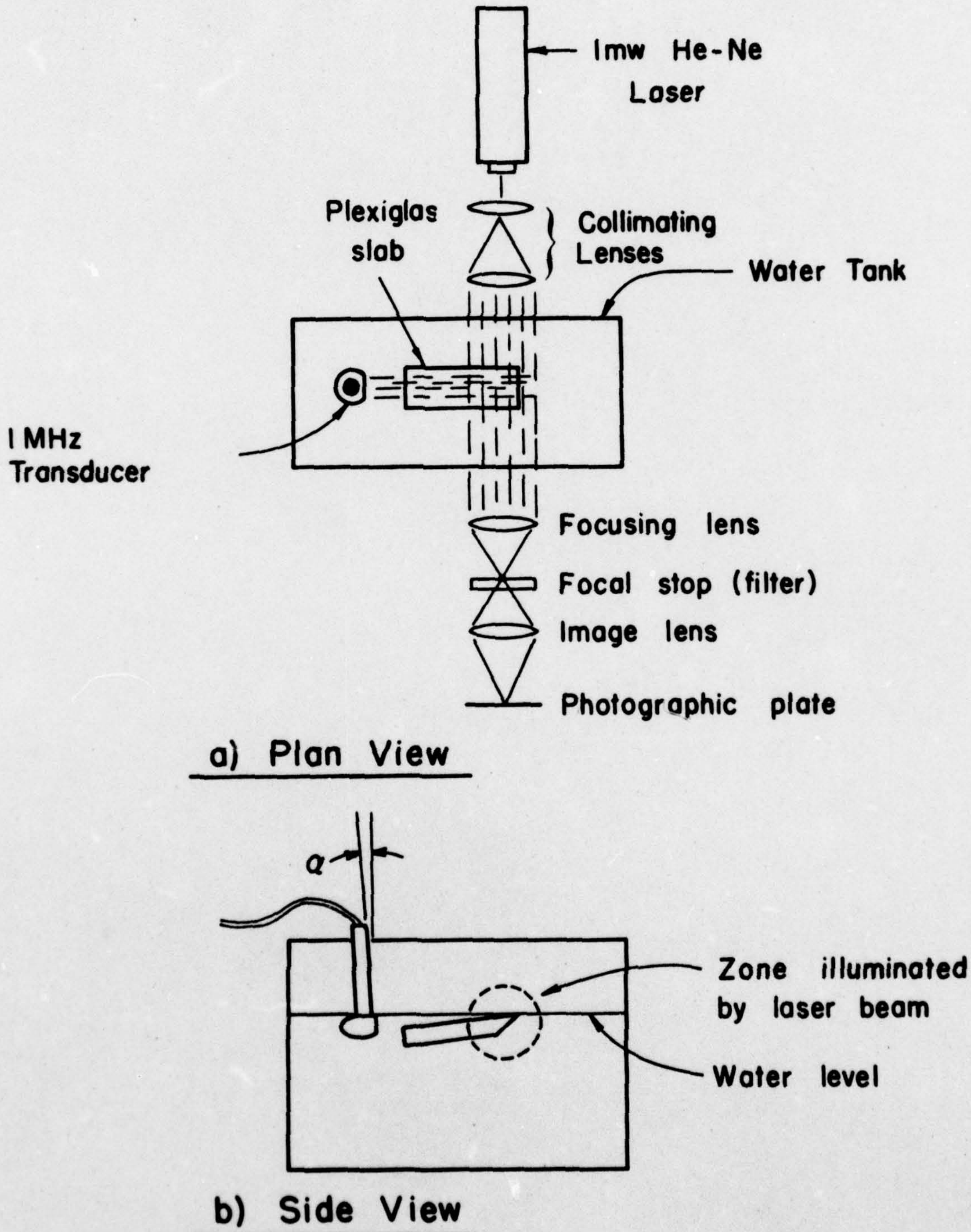

FIGURE 9. Schlieren optical arrangement.

EXPERIMENTAL RESULTS

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Power levels were varied from 15 watts to a fraction of a watt. Most of the problems encountered were associated with water surface disturbances from streaming at the higher power settings.

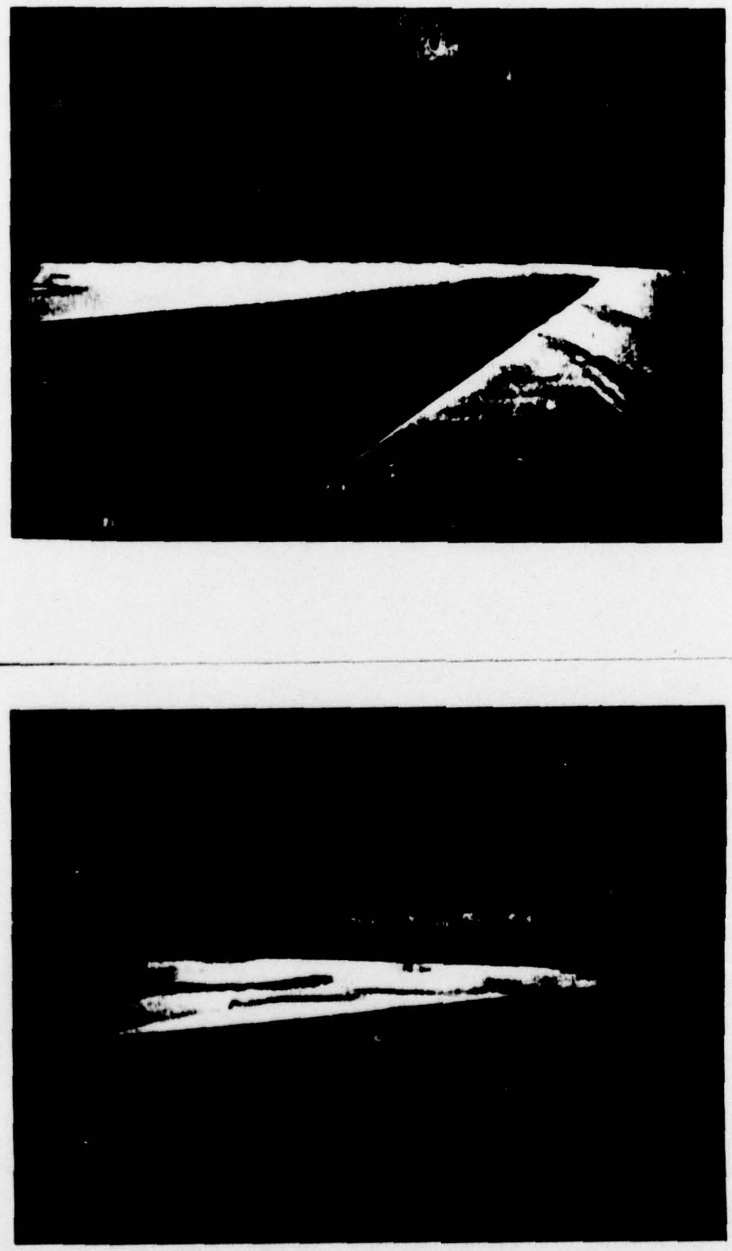

FIGURE 10. (a) Schlieren photograph of acoustic beams coupled through a plastic model. Sound is incident from the left in the waveguide which tapers to zero thickness. (b) Close up view of the modal distribution in the waveguide.

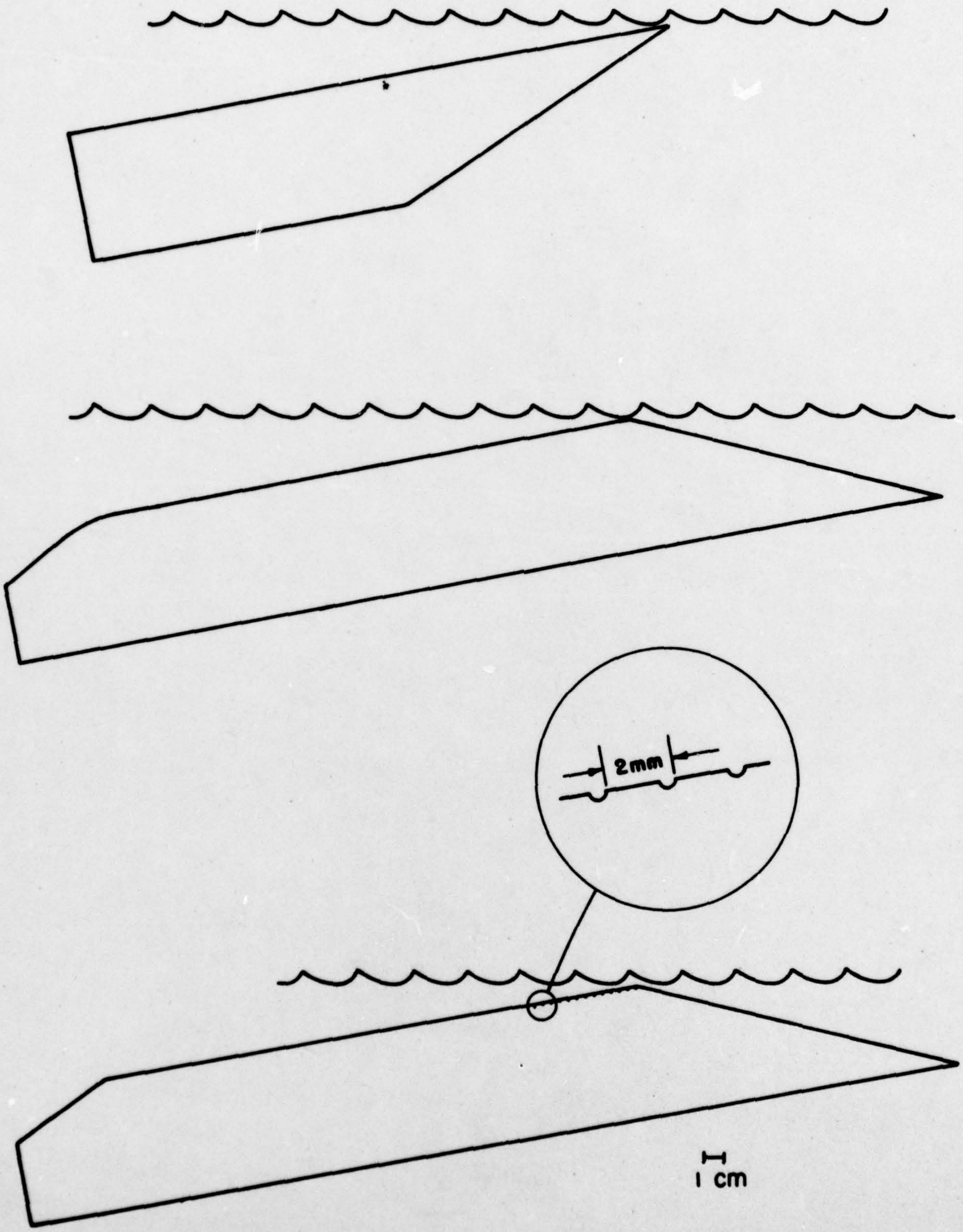

FIGURE 11. Aluminum models used for Schlieren photos. The model in the bottom drawing has a grating cut into the surface.

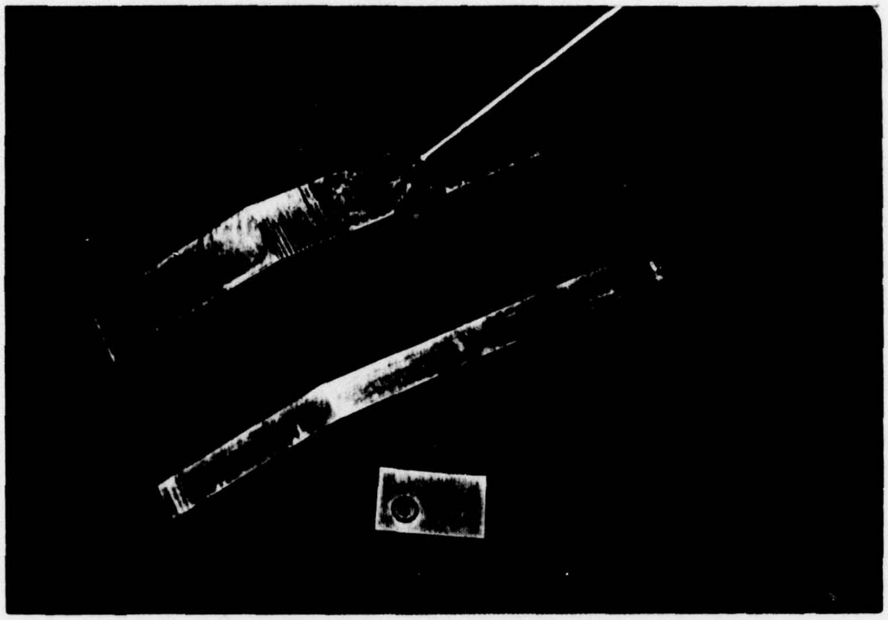

FIGURE 12. Aluminum models shown in the two bottom drawings of Figure 2.

Acoustic Field Probing

Because Schlieren techniques are generally only a qualitiative measure of acoustic field intensity the Schlieren results were extended by acoustic field probing. The first portion of the probe study was done with the models shown in Figures 11 and 12; the second probe experiment involved a completely rebuilt test tank (Figures 13-16), a 2 m. long model, and supporting hardware.

Schlieren photography indicated the (obvious) presence of acoustic energy in the waveguide above all models but was not usable to detect the coupling of energy into the solid materials. Therefore, the arrangement in Figure 17 was used to define waveguide coupling regions. An important part of the hypothesis that sound can be coupled from bound modes to beams in the substrate is that from a mode which is cut-off as guide thickness decreases goes to other propagating modes, not to the radiation spectrum. To find where sound was coupled into the model, a block was moved along the top surface while an acoustic micro probe monitored coupled energy. The data from two different runs indicate that virtually all coupling occurs in the region where water is less than the wavelength of incident 1 MHz sounds as shonw in Figure 17.

These encouraging results prompted further investigations at a more quantitative level than those proposed in the contract work statement. Because a positive indication of guided mode conversion and coupling to substrate beams had been obtained, a large model, tank, and new acoustic probes were assembled. In addition to simply demonstrating the coupling effect the new experimental setup was designed to allow quantitative measurements of field intensity and coupling efficiency. Since the tank was 2.1 m long (Figures 13-16) work with smaller beach angles was also possible. Preliminary measurements of waveguide sound intensity shown in Figure 18 indicated coupling near the shallow region of the guide. Data in this figure exhibits fluctuations resulting from the perturbation of the water surface caused by the probe motion.

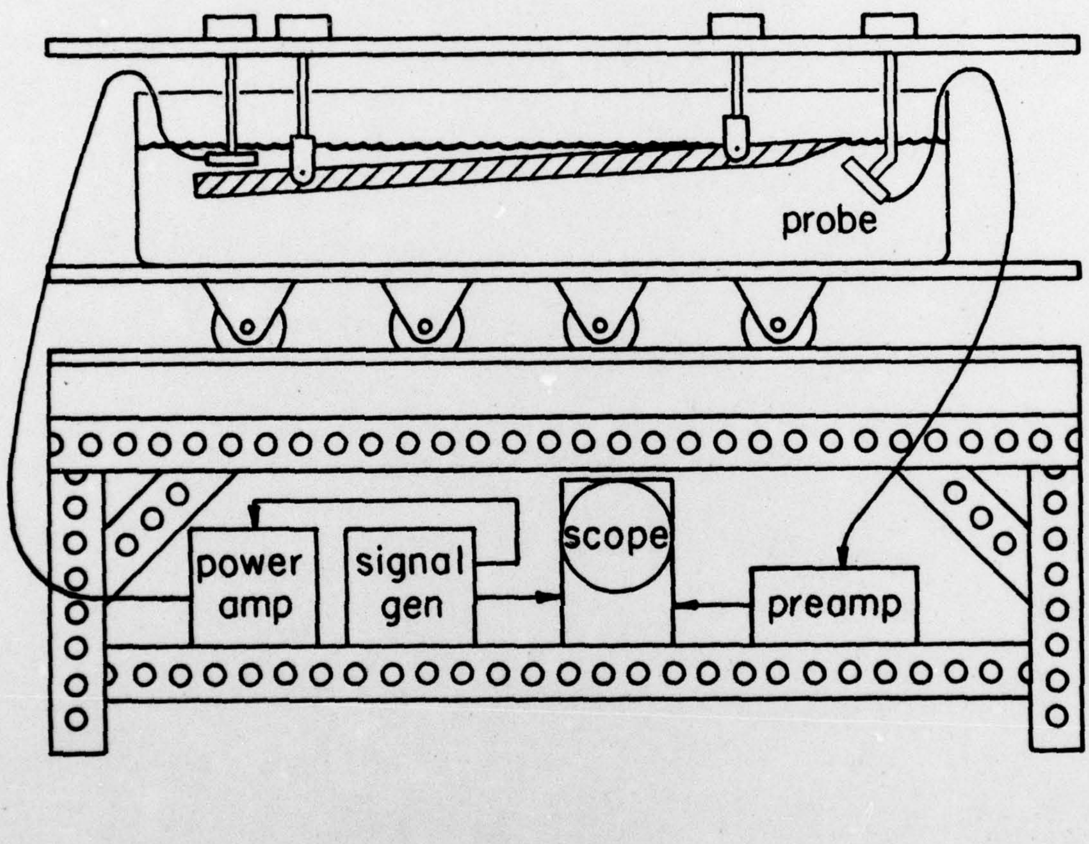

FIGURE 13. Large tank for quantitative acoustic field measuremnts with 2 meter long models. The tank supporting structure allows movement of the entire experiment for (future) Schlieren studies.

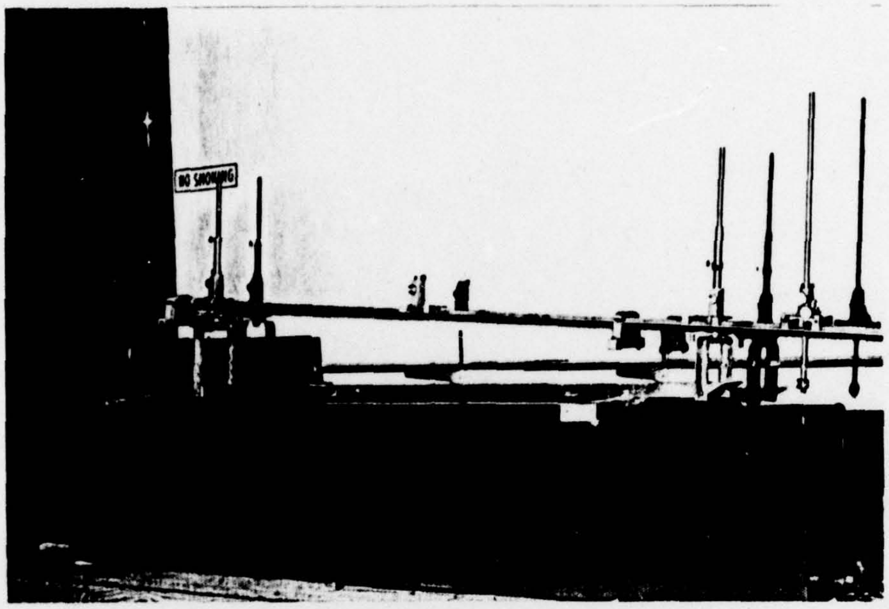

FIGURE 14. Large tank with supporting rails raised to expose aluminum model.

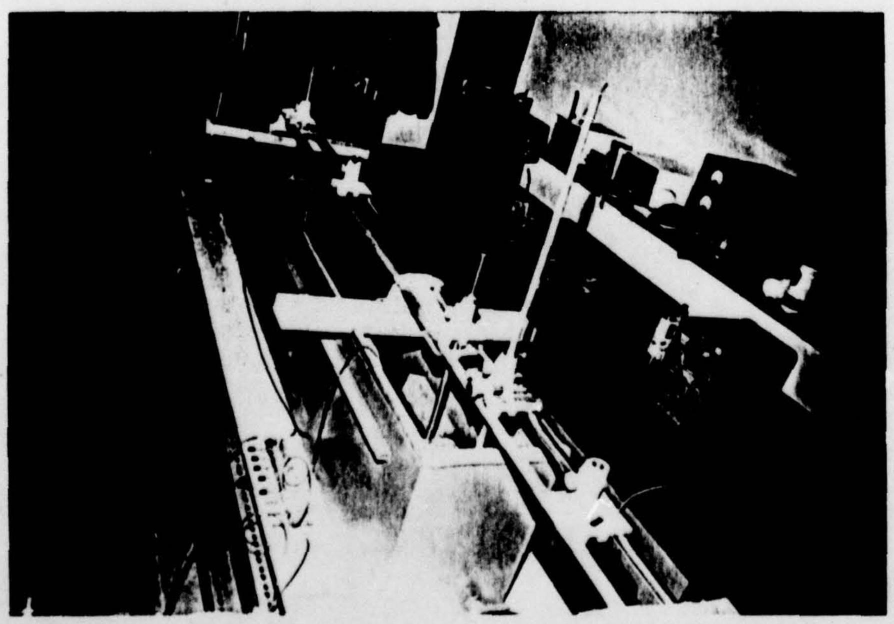

FIGURE 15. End view of large tank with model in place. Acoustic absorbing foam is visible beside the model; this material eliminates reflections from walls.

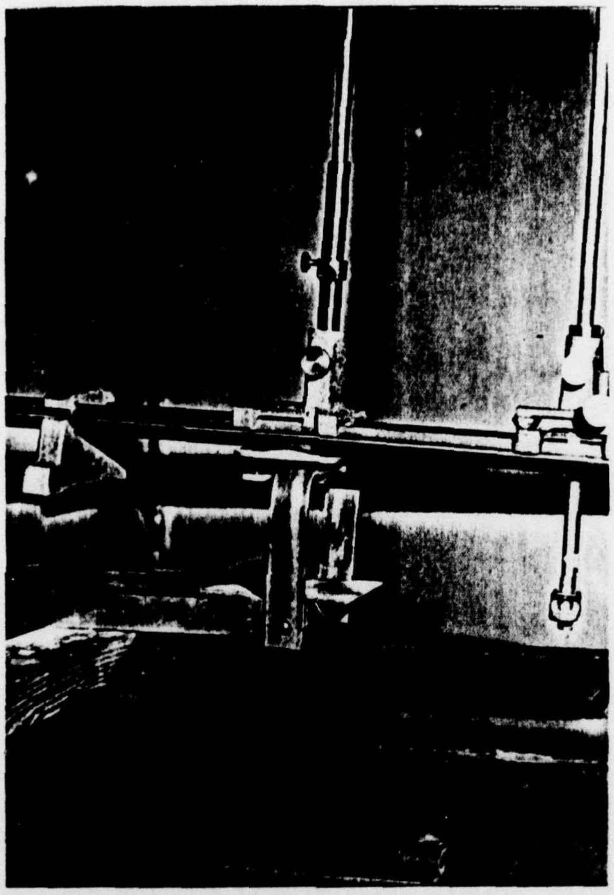

FIGURE 16. Close up view of model end. The 30^{o} milled surface on the bottom allows conversion of the radiated shear wave beams to compressional waves in the water where acoustic fields are probed.

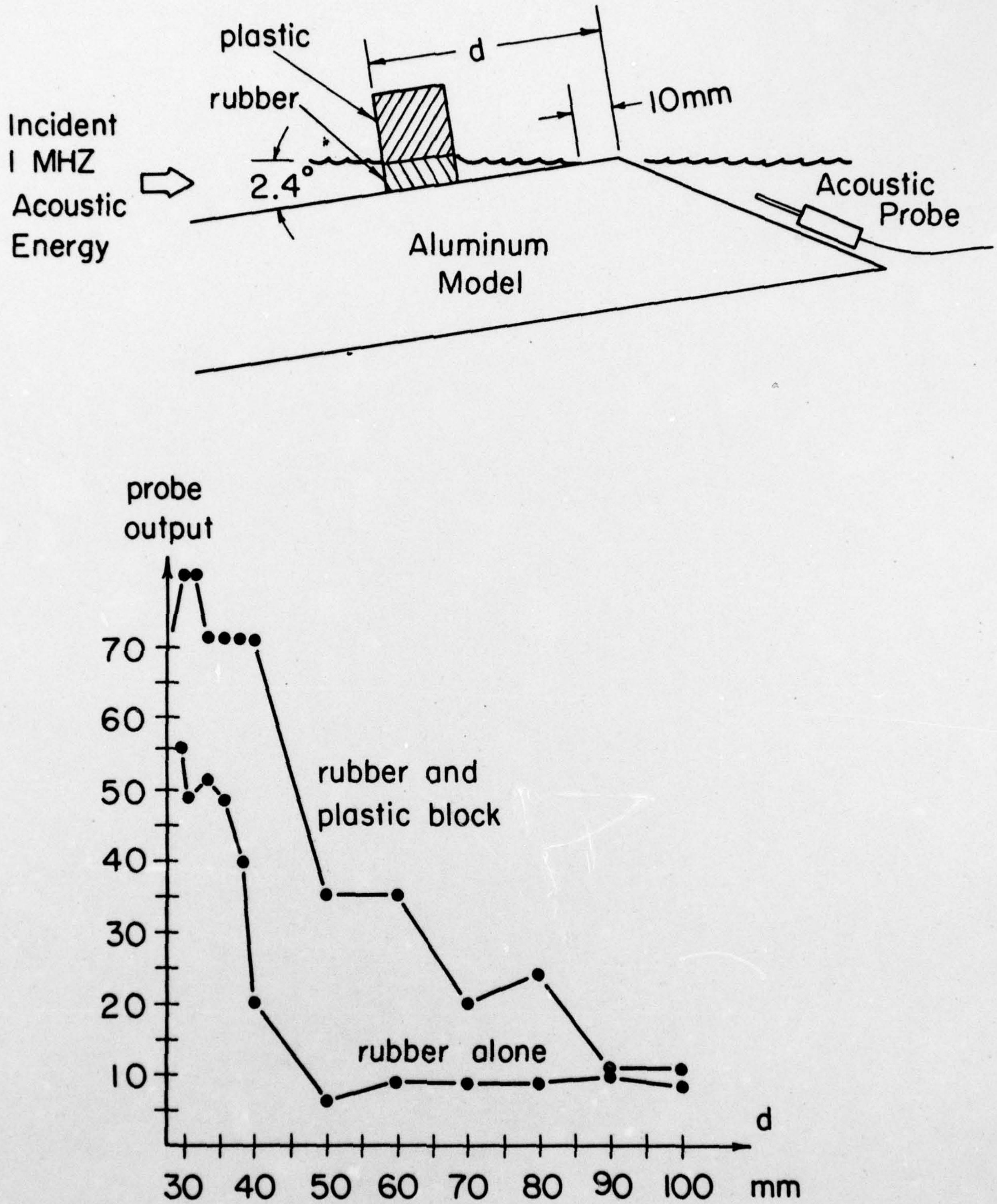

FIGURE 17. Measurements of radiated acoustic fields with guide blocked to define region of guide where coupling occurs. These data indicate that the radiated sound is coupled into the aluminum at a sharply defined water depth. This further substantiates efficient mode conversion from higher to lowest order mode.

Using a focused transducer (6 cm focal length) the acoustic field under the end of the new model was probed. Excitation was from a 2.5 MHz, 9 mm diameter flat transducer located 1.7 m from the end of the model. Figure 18 shows the experimental arrangement Figure 20 summarizes energy measurements. These data indicate a well defined (within the accuracy of measurement method) peak of output as a function of angle and position. When the strongest output ray, 20° from the normal, is traced back through the aluminum as a shear wave it indicates coupling from the "beach" area where depth is less than 0.4 mm; see Figure 21 and 22. This is the area where evaluation of the guided wave eigenvalue equation indicates cut-off for the first mode. The acoustic wavelength for these tests was 0.6 mm in water.

The measured sound amplitude in the output beam was 13 dB below a reference level established for the focused transducer 1.7 m from the exciting transducer with the aluminum model removed. The 13 dB figure includes a 6 dB estimated loss for shear to compressional wave conversion at the cut face of the aluminum model. Energy loss at the aluminum-water interface was estimated from Ewing[1] (Figure 3-4, curve 3) and an application of Snell's law to the observed output beam. This gives the 6 dB loss result. A microprobe comparison of power levels in the waveguideto output beam indicates that the output is about 6 dB down from the guide. Microprobe measurements are subject to interference within the waveguide, however, so the 6 dB comparison can only be taken to be approximate.

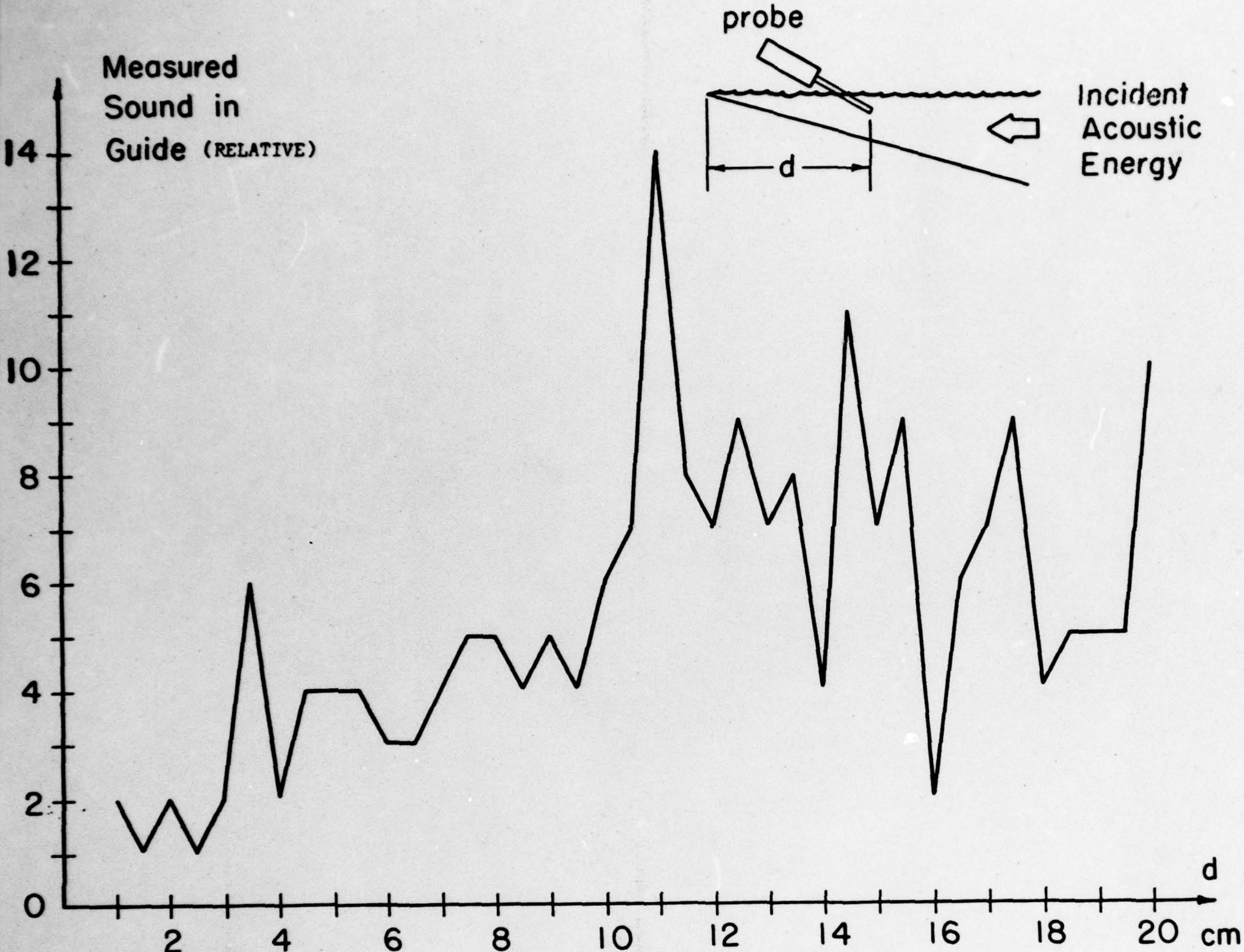

FIGURE 18. Microprobe measurement of waveguide acoustic fields. Modal interference causes a large variance in measured values; however, the general trend of coupling from the guide can be observed.

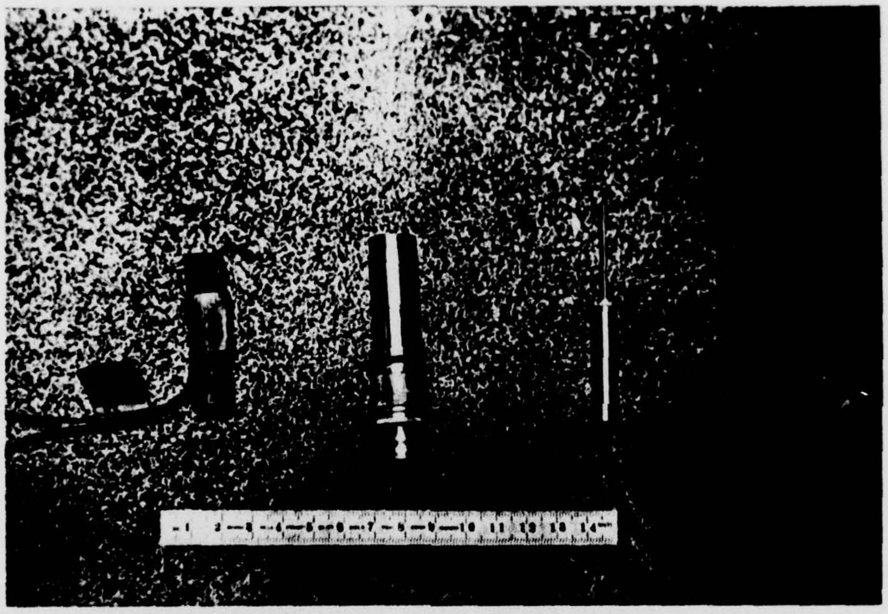

FIGURE 19. Transducers used for acoustic measurements from left to right (a) 6 cm focal length 2.5 MHz, 9mm diameter, used as a probe for output beam definition, (b) 2.5 MHz 9 mm diameter flat, used to excite 2 m long aluminum model, (c) wide band acoustic probe for measurements requiring small ($\approx$ 1 mm^2) area resolution.

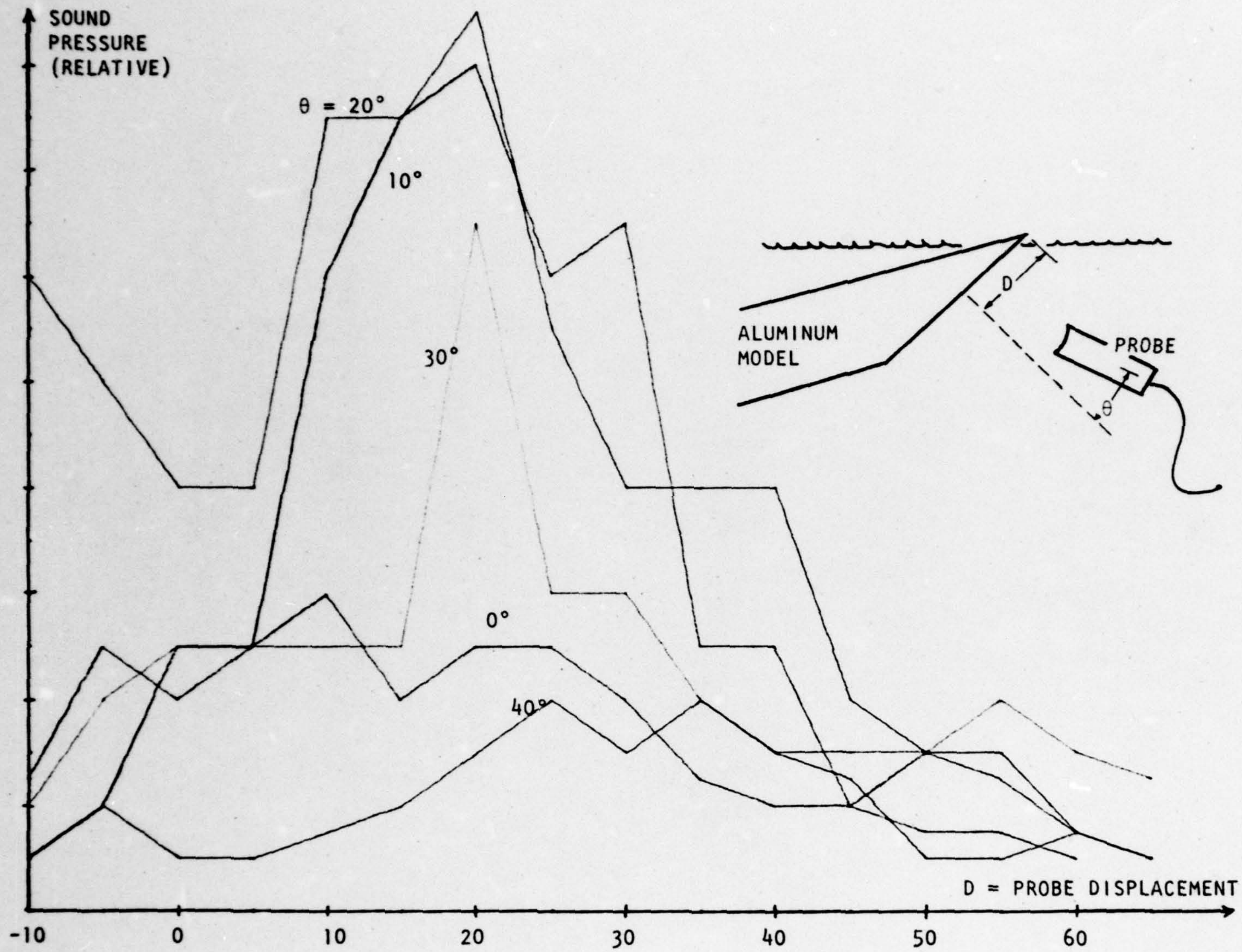

FIGURE 20. Radiated acoustic fields measured with a focused transducer. Different colors represent data taken at various transducer angles. At the maximum, received power is 13 dB below the level for a similar probe-source separation in free water. 6 dB of this loss is due to shear wave conversion at the end cut of the model.

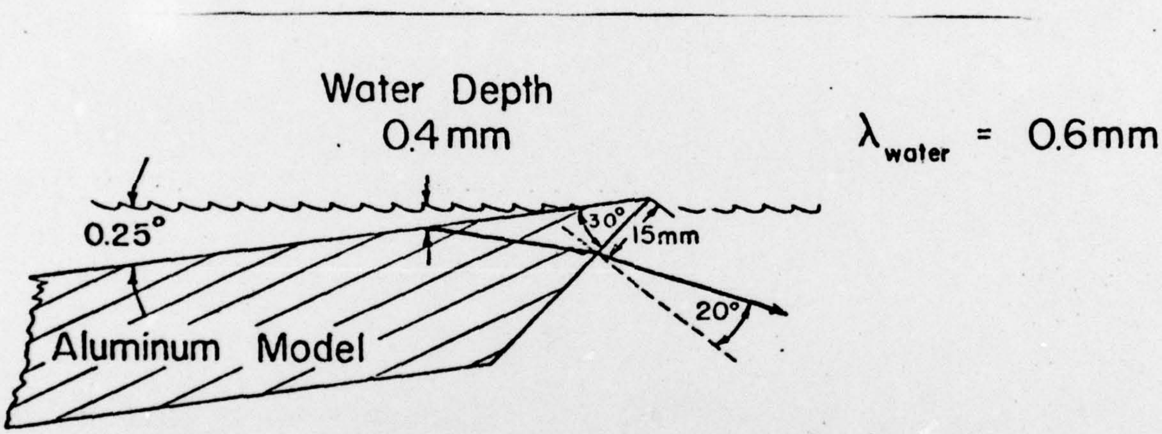

FIGURE 21. Ray trace to determine waveguide coupling region. The ray representing peak output in Figure 20 originates in the tapered waveguide where water depth is less than 0.4 mm. For this portion of the experiment, the acoustic wavelength was 0.6 mm.

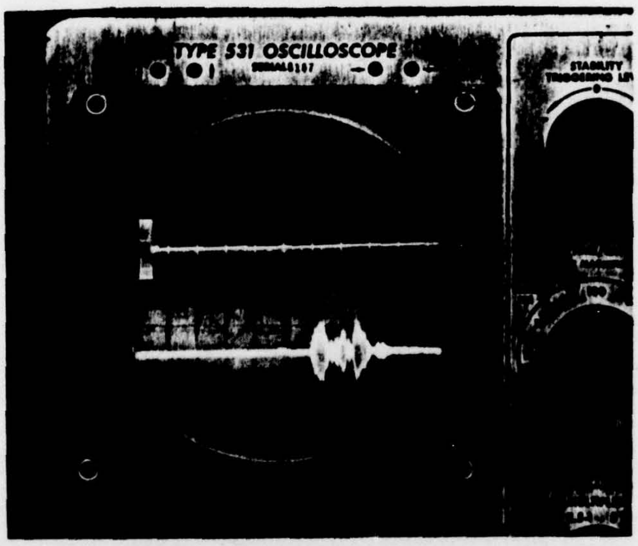

FIGURE 22. Oscilloscope display of 2.5 MHz, 50 cycle burst applied to exciting transducer (top trace) and received pulse (bottom trace). Burst excitation allows separation of direct data and signals reflected from the end of the test tank.

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

The most important results of this research may be summarized as follows:

a. There is strong evidence that acoustic waves in water entering the tapered shoreline region propagate with little attenuation until the water depth is less than the wavelength, and then couple efficiently to the shear wave in the elastic solid substrate. Experiments indicate that higher order modes in the incident acoustic wave are converted to lower order modes without appreciable reflection or radiation into the solid substrate. Because coupling into the solid occurs at shallow depth, acoustic energy arriving via acoustic ducts[7] can be expected to behave in the same manner. It appears that for materials modeling hard rock, the compressional wave is not appreciably excited since coupling to the slower shear wave starts at greater depth and is responsible for the bulk of the energy transfer.

b. The energy radiated into the solid is concentrated in a well defined beam whose angle depends on the propagation velocity of the shear wave in the solid, and on the rate of taper of the water depth.

c. Acoustic scale modeling at very large scale factors (up to 25,000/1) is a feasible and effective technique.

As outlined briefly in the introduction, certain directions for continued research are suggested by the results of this program. There is a need for solid theoretical backup for future experimental work. First, existing analyses of waves in layered media should be specialized to conform to values of the parameters of the ocean-shoreline problem. This will facilitate the calculations of propagation constants, cut-off wavelengths, field amplitudes, etc. Theoretical treatments must be initiated on the mode conversion in the tapered region, and on the radiation mechanism into the solid substrate. A possible approach to

the study of energy transfer into the solid is through a total field solution using finite difference equations. This approach could lead to the development of a computer program by which the user could readily predict the effects of practical considerations such as taper rate, constants of the media, angle of arrival of incident waves, frequency, etc.

Continued experimental work should be carried out with the objective of greater quantitative accuracy. The existing 2.1 meter tank would be entirely adequate for more precise work when upgraded with a probe carriage assembly, and automated data logging.

A final objective of continued research is to provide guidelines for future full scale experiments.

REFERENCES

1. W. M. Ewing, W. S. Jardetzky, F. Press, Elastic Waves in Layered Media, McGraw-Hill, NY (1957).

2. I. Tolstoy, C. S. Clay, Ocean Acoustics, McGraw-Hill, NY (1966).

   I. Tolstoy, "Dispersive Properties of a Fluid Layer Overlaying a Semi-infinite Elastic Solid," Bull. Seism. Soc. Am., 44, p. 493 (1954).

3. P. K. Tien, R. J. Martin, "Experiments on Light Waves in a Thin Tapered Film and a New Light Wave Coupler," Applied Physics Letters 18, p. 398 (May 1, 1971).

4. R. K. Winn, J. H. Harris, "Coupling from Multimode to Single Mode Linear Waveguides Using Horn-Shaped Structures," IEEE Trans. MTT-23, p. 92, (1975).

5. D. Marcuse, "Radiation Losses of Tapered Dielectric Slab Waveguides," Bell System Tech. J. 49, p. 273 (February 1970).

6. E. L. Hamilton, "Acoustic and Relative Properties of the Sea Floor; Shear Wave Velocity Profiles and Gradients," NUC TP 472 (July 1975).

   E. Press, "Seismic Velocities," Section 9, Handbook of Physical Constants, The Geological Society of America, Memoir 97 (1966).

7. C. L. Bartberger, L. L. Ackler, "Normal Mode Solutions for 2 and 3-Layer Velocity Profiles With Bottom" (U) Confidential, U.S.N. Journal of Underwater Acoustics 19, p. 67 (January ,1969).

   M.A. Pederson, D. F. Gordon, "Normal Mode Theory Applied to Short-range Propagation in an Underwater Acoustic Surface Duct," J. Acoust. Soc. Am. 37, p. 105, (January 1965).